

**M E T U**  
**Department of Mathematics**

Introduction to Differential Equations				
Final				
Code	: <i>Math 219</i>	Last Name	:	
Acad. Year	: <i>2016-2017</i>	Name	:	Student No. :
Semester	: <i>Spring</i>	Department	:	Section :
Coordinator	: <i>Özgür Kişisel</i>	Signature	:	
Date	: <i>June.9.2017</i>	4 QUESTIONS ON 4 PAGES		
Time	: <i>09:30</i>	TOTAL 100 POINTS		
Duration	: <i>120 minutes</i>			
1	2	3	4	SHOW YOUR WORK

Question 1 (25 pts) Solve the system

$$\mathbf{x}' = \begin{bmatrix} 5 & 1 & -4 \\ 4 & 3 & -5 \\ 3 & 1 & -2 \end{bmatrix} \mathbf{x}$$

Question 2 (25 pts) Consider the ODE

$$x^2 y'' - 2xy' + 2y = 2e^x x^3, \quad x > 0.$$

- a) Find all solutions of the corresponding homogeneous equation.  
 b) Find all solutions of the equation by using variation of parameters.

Question 3 (25 pts) Find the solution of the IVP

$$y'' + y' + \frac{5}{4}y = 1 - u_\pi(t)$$

where  $y(0) = 1$ ,  $y'(0) = -1$ .

Question 4 (25 pts) Solve the heat conduction problem

$$\begin{aligned} 9u_{xx} &= (1 + t^2)u_t, & 0 < x < \pi, & t > 0 \\ u(0, t) &= 0, & u_x(\pi, t) &= 0 \\ u(x, 0) &= x \end{aligned}$$

What is the steady state solution, i.e.  $\lim_{t \rightarrow \infty} u(x, t)$ ?